Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

Spectral Density of Cooper Pairs in Series Double Coupled Quantum Dots - Superconductor Josephson Junction

Dr. Rajendra Kumar
Associate Professor, Department of Physics, Sanatan Dharam Mahavidyalaya, Muzaffarnagar, Uttar Pradesh, INDIA.

Corresponding Author: rkphddpt@gmail.com

www.sjmars.com || Vol. 3 No. 5 (2024): October Issue

Date of Submission: 16-10-2024	**Date of Acceptance:** 21-10-2024	**Date of Publication:** 28-10-2024

ABSTRACT

We study the simple device of two laterally-coupled quantum dots, where one of quantum dot Connected with left superconductor lead and also connected right superconductor lead (QD1) and (QD2) are coupling to each other as Series quantum dots. The Spectral Density and Josephson current through this double quantum dots junction and formed between superconducting leads having s-wave symmetry of the superconducting order parameter is analysed. For this purpose, we have considered a renormalized Anderson model that includes the double coupled of the superconducting leads with quantum dots level and an attractive BCS-type effective interaction in superconducting leads. We have calculated the double-particle spectral function and Josephson current of the quantum dot, theoretical analysis.

Keywords- Spectral Density, Cooper Pairs, Double Quantum Dots, Josephson Junction, Superconductivity.

I. INTRODUCTION

In the recent past advancement in materials fabrication at nano-scale, made it possible to realize superconducting quantum dot nanoscopic Josephson junctions having their potential technological applications in electronic devices. The quantum dots (QD) are semiconductor nanoscopic structures, where electrons are confined to zero dimensions and due to the quantum confinement these QD have discrete energy levels like an atom [1,3]. Therefore, the QD is a class of nano-materials, having discrete energy levels and the possibility to tune the separation between energy levels and on dot Coulomb energy by changing the size, shape and environment of the QD. In superconducting materials, the electronic states close to the Fermi level are the bound electron Cooper pairs and electrons of Cooper pair can tunnel coherently through the discrete energy levels of quantum dot serving as a barrier in (S-QD-S) junction [3,4]. The QD believed to have potential applications in quantum devices like quantum computers and quantum communication and knowledge of the electronic properties of QD's interfaced with variety of environments is an important step in this direction. The superconducting–QD tunnel junctions are obtained by coupling a QD with superconducting leads, and provide a way to study influence of separation of dot energy levels, coupling parameter between dot states and superconducting leads and Coulomb correlations on quantum transport in these (S-QD-S) Josephson Junction .Recently, the influence of the electron correlations on the Josephson transport across superconductor quantum dot superconductor (S-QD-S) junction have been investigated.

One of the interesting issues in quantum transport in nano-scopic materials is the understanding of Josephson electronic current through QD interfaced with superconducting leads [5,8] due to immense technological potential. There have been several attempts to study the electronic transport through correlated QD sandwich between the superconducting leads [9, 11]. In these studies, an interplay of the single particle and Josephson Cooper pair tunnelling on supercurrent across the superconducting quantum dot junction has been analyzed [19-27]. It is pointed out that the Josephson super current across (S-QD-S) junction depend on the competitive role of the single particle tunnelling and the energy of the dot level with respect to binding energy of the Cooper pairs in superconducting leads and also on the Josephson Cooper pair tunnelling process across (S-QD-S) junction. In these studies a single level quantum dot is considered as a sandwich between superconducting leads having s-wave pairing symmetry [28-30]. At low temperatures, Cooper pairs from one side of superconducting lead get tunnel to other side one -by-one through the discrete energy level of QD without any pair breaking effects as coherence length of superconducting state in conventional superconductors is much larger compared to the size of quantum dots nanostructure. On the other hand, there are studies extended to a double quantum dots in series configurations coupled to superconducting leads [27,28] (S-DQD-S junction). The S-DQD-S double quantum dots Josephson junction exhibit rich physics as compared to S-QD-S single quantum dot tunnel junction and provide an opportunity to study theoretically the electronic properties within two impurity extended Anderson model system. Electronic transport through a series double quantum dots (DQD) coupled to superconducting leads within numerical renormalization group approach has been analyzed and transmission probability for the system of two coupled identical quantum dots studied [27]. For the case of double quantum dots having inter dot hopping along-with on dot Coulomb repulsion, the conductance through a half-filled double quantum dot thoroughly studied and relation between electronic transport and electron-configuration of double quantum dots have also been analyzed [28]. There are theoretical attempts even for the array of multiple quantum dots placed in parallel configuration between two electron reservoirs in normal state and coupled with the quantum dots through time-dependent tunnelling matrix elements [29].

Recently, the attempts have also been made to analyze two serially aligned quantum dots with large tunnel coupling with the BCS superconductors as leads. It is further pointed out that the electronic hopping between the dots states is strong compared to the coupling between superconductor dots states [31] and play important role in understanding the electronic structure and nature of Josephson transport in the coupled double quantum dots tunnel junctions [32]. The electronic structure of a double quantum dots device with both the dot coupled to perfect conducting leads strongly influence the local density of states on the dots and there by electronic transport behaviour across the S-DQD-S system. Recently, within Keldysh non-equilibrium Green's function an analysis of local electronic density of states and Josephson current (i.e. electronic spectral density and current) has also been attempted for multiple dots [33].

Further, it is interesting to pointed out that the tunnelling conductance between a metallic electrode and strongly correlated material can be connected with effective local density of states and Josephson current in these tunnel junctions defined in terms of integrated single particle spectral function at Fermi level. The influence of coupled quantum dots sand-witched between normal as well as BCS superconducting leads on the electronic transport across such system is not clearly understood so far experimentally as well as theoretically. Therefore, in the present work, we have planned to analyze the spectral density of Cooper pair as a function of various coupling parameters in double T-shaped coupled quantum dots connected to conventional superconducting leads with s-wave pairing symmetry. This analysis is important as electronic spectral density of Cooper pair at Fermi level represents the zero bias conductance across such S-DQD-S tunnel junction and provide an interesting insight into the electronic structure and Josephson transport behaviour in nano-junctions [33, 34]. In the next section, we have presented our model Hamiltonian and theoretical formulation for the calculations and analysis of spectral density in S-DQD-S Josephson tunnel junction.

II. THEORETICAL FORMULATION

Two Quantum Dots in series with connected to QD1 with left Superconductor leads & QD2 also connected right superconductor leads. QD1 & QD2 are coupling to each other so the Hamiltonian with series Quantum Dots Fig (1).
The model Hamiltonian for our (S-QD-S) system can be described as follows:

$$\mathrm{H} = \mathrm{H}_D + \sum_{i=1,2}\left(\mathrm{H}_i + \mathrm{H}_t\right) \quad \text{.......... (1)}$$

Where,

$$\mathrm{H}_D = \sum_{\sigma}\left(\varepsilon_1 d^+_{1\sigma} d_{1\sigma} + \varepsilon_2 d^+_{2\sigma} d_{2\sigma}\right) + U_1 n^d_{1\uparrow} n^d_{1\downarrow} + U_2 n^d_{2\uparrow} n^d_{2\downarrow} \quad \text{.......... (1a)}$$

$$\mathrm{H}_i = \sum_{\mathrm{k}=\uparrow\downarrow,\eta=L,R} \varepsilon_k c^+_{\eta k\sigma} c_{\eta k\sigma} - \sum_k\left(\Delta_{\eta=L,R} c^+_{\eta k\sigma} c^+_{\eta-k\downarrow} + \Delta^+_{\eta=L,R} c_{\eta-k\downarrow} c_{\eta k\uparrow}\right) \quad \text{.......... (1b)}$$

$$\mathrm{H}_t = t_d\left(d^+_{1\sigma} d_{2\sigma} + d^+_{2\sigma} d_{1\sigma}\right) + \sum_{k\sigma} T_{k1}\left(c^+_{1k\sigma} d_{1\sigma} + d^+_{1\sigma} c_{1k\sigma}\right) + \sum_{k\sigma} T_{k2}\left(c^+_{2k\sigma} d_{2\sigma} + d^+_{2\sigma} c_{2k\sigma}\right) \quad \text{.......... (1c)}$$

Where, H_d (Eq.1a) is the Hamiltonian for correlated QD the single energy level (ε_1) and Double energy level (ε_2)Which have $\langle \eta^d_{1\uparrow} \rangle = \langle \eta^d_{1\downarrow} \rangle = \langle \eta^d_{2\uparrow} \rangle = \langle \eta^d_{2\uparrow} \rangle = \langle \eta \rangle$,Where $\langle \eta \rangle$ is probability occupancy on the dot. In (Eq.1a) U_1 and U_2 is the dots Coulomb energy. H_i (Eq1b) is the BCS Hamiltonian for left (η=1) and right (η=2) side superconductor. In H_η the

Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

first term represents the attractive interaction between the electrons of superconducting lead responsible to from Cooper pairs and yield a BCS superconducting state described by a gap at the Fermi level. (Eq.1c) represents the possibility of the dot tunnelling between QD1 and QD2, QD1 connected left and QD2 right superconducting leads and QD state and vice-versa. The $c_{\eta k\sigma}\left(c^{+}_{\eta k\sigma}\right)$ represents the annihilation (creation) operators for the superconducting lead $d_{\sigma}\left(d^{+}_{\sigma}\right)$ represents the annihilation (creation) operators for the dot state.

To study the electronic transport behaviour of S-QD-S junction, we start with the following Green's function: $G_{11}=\left\langle\left\langle c_{1k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle$.The equation of motion corresponding to above Green's function can be obtained as:

$$\omega\left\langle\left\langle c_{1k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\frac{1}{2\pi}\left\langle\left[c_{1k\uparrow};c^{+}_{1k\uparrow}\right]\right\rangle+\left\langle\left\langle\left[c_{1k\uparrow},\mathrm{H}\right];c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (2)$$

Solving commutator $\left[c_{1k\uparrow},\mathrm{H}\right]$; we get higher order Green's function. To linearize higher order Green's function in to the lower ones, we follow a mean field approximation [6-7] and get following twelve coupled Green's function equations of motion:

$$(\omega-\varepsilon_k)\left\langle\left\langle c_{1k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\frac{1}{2\pi}-\Delta_1\left\langle\left\langle c^{+}_{1-k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle+V_1\left\langle\left\langle d_{1\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (3)$$

$$(\omega+\varepsilon_k)\left\langle\left\langle c^{+}_{1-k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\Delta^{+}_1\left\langle\left\langle c_{1k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle-V_1\left\langle\left\langle d^{+}_{1\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (4)$$

$$(\omega-\varepsilon)\left\langle\left\langle d_{1\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=t_d\left\langle\left\langle d_{2\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle+V_1\left\langle\left\langle c_{1k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (5)$$

$$(\omega+\varepsilon)\left\langle\left\langle d^{+}_{1\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=-t_d\left\langle\left\langle d^{+}_{2\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle-V_1\left\langle\left\langle c^{+}_{1k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (6)$$

$$(\omega-\varepsilon)\left\langle\left\langle d_{2\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=t_d\left\langle\left\langle d_{1\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle+V_2\left\langle\left\langle c_{2k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (7)$$

$$(\omega+\varepsilon)\left\langle\left\langle d^{+}_{2\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=-t_d\left\langle\left\langle d^{+}_{1\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle-V_2\left\langle\left\langle c^{+}_{2k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (8)$$

$$(\omega+\varepsilon_k)\left\langle\left\langle c^{+}_{1k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\Delta^{+}_1\left\langle\left\langle c_{1-k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle-V_1\left\langle\left\langle d^{+}_{1\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (9)$$

$$(\omega-\varepsilon_k)\left\langle\left\langle c_{2k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=-\Delta_2\left\langle\left\langle c^{+}_{2-k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle+V_2\left\langle\left\langle d_{2\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (10)$$

$$(\omega+\varepsilon_k)\left\langle\left\langle c^{+}_{2k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\Delta^{+}_2\left\langle\left\langle c^{+}_{2k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle-V_2\left\langle\left\langle d^{+}_{2\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (11)$$

$$(\omega-\varepsilon_k)\left\langle\left\langle c_{1-k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=-\Delta_1\left\langle\left\langle c^{+}_{1k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle+V_2\left\langle\left\langle d_{1\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (12)$$

$$(\omega+\varepsilon_k)\left\langle\left\langle c^{+}_{2-k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\Delta^{+}_2\left\langle\left\langle c_{2k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle-V_2\left\langle\left\langle d^{+}_{2\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (13)$$

$$(\omega-\varepsilon_k)\left\langle\left\langle c^{+}_{2k\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=-\Delta_2\left\langle\left\langle c^{+}_{2k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle+V_2\left\langle\left\langle d_{2\uparrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle \quad (14)$$

Where, $\Delta_1=\sum_k U\left\langle c_{1-k\downarrow};c_{1k\uparrow}\right\rangle$ is the superconducting order parameter of the left side ($\eta=1$) superconductor and $\Delta_2=\sum_k U\left\langle c_{2-k\downarrow};c_{2k\uparrow}\right\rangle$ is the superconducting order parameter of the right side ($\eta=2$) superconductor. Here, we assume that both superconductors are identical and have same superconducting order parameter (i.e. $\Delta_1=\Delta_2=\Delta$) & ($V_1=V_2=V$). Solving the above coupled equations (3-14) a simple algebra leads to the final expression for the desired Green's function as [20-27]:

Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

$$\left\langle\left\langle c^{+}_{1-k\downarrow};c^{+}_{1k\uparrow}\right\rangle\right\rangle=\frac{1}{2\pi}\left[\frac{\left\{\begin{array}{l} t_d^2\left(a_k\omega_k^7+b_k\omega_k^5+c_k\omega_k^4+d_k\omega_k^3+e_k\omega_k^2+f\omega_k\right)+a_k^*\omega_k^9+b_k^*\omega_k^7+\\ c_k^*\omega_k^6+d_k^*\omega_k^5+e_k^*\omega_k^4+f_k^*\omega_k^3+g_k^*\omega_k^2+h_k^*\omega_k\end{array}\right\}}{\left\{\begin{array}{l} t_d^2\left(a_k^1\omega_k^8+b_k^1\omega_k^6+c_k^1\omega_k^4+d_k^1\omega_k^2+e_k^1\omega_k+f_k^1\right)+a_k^2\omega_k^{10}+b_k^2\omega_k^8+\\ c_k^2\omega_k^6+d_k^2\omega_k^5+e_k^2\omega_k^4+f_k^2\omega_k^3+g_k^2\omega_k^2+h_k^2\omega_k+i_k^2\end{array}\right\}}\right] \quad \text{..... (15)}$$

Where,

$a_k=(2+4|\Delta|)$, $b_k=(2\varepsilon^2-4\varepsilon_k^2+|\Delta|\varepsilon_k+V^2+2)$, $c_k=\left(4|\Delta|\varepsilon^2-4|\Delta|\varepsilon_k^2+V^2\varepsilon_k-V^2\varepsilon+2\varepsilon-4\varepsilon_k\right)$,

$d_k=\left(2\varepsilon_k^4-4\varepsilon^2\varepsilon_k^2+2|\Delta|^2+4|\Delta|\varepsilon^2\varepsilon_k-4|\Delta|\varepsilon_k^3-V^2\varepsilon_k^2-V^2\varepsilon\varepsilon_k+2\varepsilon_k-2\varepsilon\varepsilon_k+2\varepsilon\varepsilon_k^3+3V^2|\Delta|\right)$,

$e_k=\left(4|\Delta|\varepsilon^2\varepsilon_k^2+V^2\varepsilon\varepsilon_k^2-V^2\varepsilon_k^3+2\varepsilon_k^3-2\varepsilon\varepsilon_k^2+V^2|\Delta|\varepsilon-V^2|\Delta|\varepsilon_k\right)$,

$f_k=\left(2\varepsilon^2\varepsilon_k^4+2|\Delta|^2\varepsilon^2-4|\Delta|\varepsilon^2\varepsilon_k^2+V^2\varepsilon\varepsilon_k^3+2\varepsilon\varepsilon_k^3-3V^2|\Delta|\varepsilon\varepsilon_k\right)$,

$a_k^*=1$, $b_k^*=\left(-2\varepsilon^2-2\varepsilon_k^2+2|\Delta|-3V^2\right)$, $c_k^*=\left(-2V^2\varepsilon-V^2\varepsilon_k\right)$,

$d_k^*=\left(\varepsilon^4+\varepsilon_k^4 4\varepsilon^2\varepsilon_k^2+|\Delta|^2-4|\Delta|\varepsilon^2-2|\Delta|\varepsilon_k^2+3V^2\varepsilon^2+3V^2\varepsilon_k^2-3V^2\varepsilon\varepsilon_k-3|\Delta|V^2\right)$,

$e_k^*=\left(-3V^2\varepsilon^3-V^2\varepsilon_k^3-V^2\varepsilon\varepsilon_k^2-V^2\varepsilon^2\varepsilon_k+|\Delta|V^2\varepsilon+|\Delta|V^2\varepsilon_k\right)$,

$f_k^*=\left(-2\varepsilon^2\varepsilon_k^4-2\varepsilon_k^2\varepsilon^4-2|\Delta|^2\varepsilon^2+2|\Delta|\varepsilon^4+4|\Delta|\varepsilon^2\varepsilon_k^2+3V^2\varepsilon\varepsilon_k^3-3V^2\varepsilon^2\varepsilon_k^2+3V^2\varepsilon^3\varepsilon_k+3|\Delta|V^2\varepsilon^2-3|\Delta|V^2\varepsilon\varepsilon_k\right)$,

$g_k^*=\left(V^2\varepsilon^2\varepsilon_k^3+V^2\varepsilon^3\varepsilon_k^2-|\Delta|V^2\varepsilon^3-|\Delta|V^2\varepsilon^2\varepsilon_k\right)$,

$h_k^*=\left(\varepsilon^4\varepsilon_k^4+|\Delta|^2\varepsilon^4-2|\Delta|\varepsilon^4\varepsilon_k^2-3V^2\varepsilon^3\varepsilon_k^3+|\Delta|V^2\varepsilon^3\varepsilon_k+2|\Delta|V^2\varepsilon^3\varepsilon_k\right)$,

$a_k^1=(-2)$, $b_k^1=\left(4V^2+6\varepsilon_k^2-2\varepsilon^2-6|\Delta|\right)$,

$c_k^1=\left(-4V^2\varepsilon\varepsilon_k-8\varepsilon_k^2V^2+6|\Delta|V^2-6\varepsilon_k^4+6\varepsilon^2\varepsilon_k^2-6|\Delta|^2+12\varepsilon_k^2|\Delta|-6|\Delta|\varepsilon^2\right)$,

$d_k^1=\left(4V^2\varepsilon_k^2+8V^2\varepsilon\varepsilon_k^3+4|\Delta|^2V^2-6|\Delta|V^2\varepsilon_k^2-6|\Delta|V^2\varepsilon\varepsilon_k-6\varepsilon^2\varepsilon_k^4-6|\Delta|^2\varepsilon^2+6|\Delta|^2\varepsilon_k^2-6|\Delta|\varepsilon^4+12|\Delta|\varepsilon^2\varepsilon_k^2\right)$,

$e_k^1=\left(-2|\Delta|V^2\varepsilon_k^3-2|\Delta|V^2\varepsilon\varepsilon_k^2\right)$,

$f_k^1=\left(-4\varepsilon\varepsilon_k^5V^2-4|\Delta|^2V^2\varepsilon\varepsilon_k+6|\Delta|V^2\varepsilon\varepsilon_k^3+2\varepsilon^2\varepsilon_k^6+6|\Delta|^2\varepsilon^2\varepsilon_k^2-6|\Delta|\varepsilon^2\varepsilon_k^4-2|\Delta|^3\varepsilon^2\right)$,

$a_k^2=1$, $b_k^2=\left(-4V^2-3\varepsilon_k^2-2\varepsilon^2+3|\Delta|\right)$,

$c_k^2=\left(8V^2\varepsilon_k^2+4\varepsilon^2V^2-4V^2\varepsilon\varepsilon_k+3\varepsilon_k^4+\varepsilon^4+6\varepsilon^2\varepsilon_k^2+3|\Delta|^2-6|\Delta|\varepsilon_k^2-6|\Delta|\varepsilon^2\right)$,

$d_k^2=\left(-8|\Delta|V^2\varepsilon_k-4|\Delta|V^2\varepsilon\right)$,

$$e_k^2=\left(\begin{array}{l}-4\varepsilon_k^4V^2+4V^2\varepsilon^3\varepsilon_k-8V^2\varepsilon^2\varepsilon_k^2+8V^2\varepsilon\varepsilon_k^3-4|\Delta|^2V^2-\varepsilon^2-\varepsilon\varepsilon_k\\ -\varepsilon_k^6-3\varepsilon^4\varepsilon_k^2-6\varepsilon^2\varepsilon_k^4-6|\Delta|^2\varepsilon^2-3|\Delta|^2\varepsilon_k^2+3|\Delta|\varepsilon_k^4+3|\Delta|\varepsilon^4+12|\Delta|\varepsilon^2\varepsilon_k^2+|\Delta|^2\end{array}\right),$$

$f_k^2=\left(8|\Delta|V^2\varepsilon_k^3+8|\Delta|V^2\varepsilon^3+4|\Delta|V^2\varepsilon^2\varepsilon_k+4|\Delta|V^2\varepsilon\varepsilon_k^2\right)$,

$$g_k^2=\left(\begin{array}{l}-8V^2\varepsilon^3\varepsilon_k^3+4V^2\varepsilon^2\varepsilon_k^4-4V^2\varepsilon\varepsilon_k^5+4|\Delta|^2V^2\varepsilon^2-2|\Delta|^2V^2\varepsilon\varepsilon_k-4|\Delta|V^2\varepsilon^2\varepsilon_k^2+4|\Delta|V^2\varepsilon\varepsilon_k^3+\\ \varepsilon^4\varepsilon_k^4+2\varepsilon^2\varepsilon_k^6+3|\Delta|^2\varepsilon^4+6|\Delta|^2\varepsilon^2\varepsilon_k^2-6|\Delta|\varepsilon^4\varepsilon_k^2-6|\Delta|\varepsilon^2\varepsilon_k^4-2|\Delta|^2\varepsilon^2\end{array}\right),$$

$h_k^2=\left(-8|\Delta|V^2\varepsilon^3\varepsilon_k^2-4|\Delta|V^2\varepsilon^2\varepsilon_k^3\right)$,

Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

$$i_k^2 = \left(4V^2\varepsilon^3\varepsilon_k^5 + 4|\Delta|^2 V^2\varepsilon^3\varepsilon_k - \varepsilon^4\varepsilon_k^6 - 3|\Delta|^2\varepsilon^4\varepsilon_k^2 + 3|\Delta|\varepsilon^4\varepsilon_k^4 + |\Delta|^2\varepsilon^4\right)$$

Where,

ω1, ω2, ω3, ω4, ω5, ω6, ω7, ω8, ω9, and ω10, are ten poles. There, poles are calculated with the help of **MATLAB (R2008b) technique;**

$$A_{1k} = \left[\frac{t_d^2\left(a_k\omega_k^7 + b_k\omega_k^5 + c_k\omega_k^4 + d_k\omega_k^3 + e_k\omega_k^2 + f\omega_k\right) + a_k^*\omega_k^9 + b_k^*\omega_k^7 + c_k^*\omega_k^6 + d_k^*\omega_k^5 + e_k^*\omega_k^4 + f_k^*\omega_k^3 + g_k^*\omega_k^2 + h_k^*\omega_k}{(\omega_{1k}-\omega_{2k})(\omega_{1k}-\omega_{3k})(\omega_{1k}-\omega_{4k})(\omega_{1k}-\omega_{5k})(\omega_{1k}-\omega_{6k})(\omega_{1k}-\omega_{7k})(\omega_{1k}-\omega_{8k})(\omega_{1k}-\omega_{9k})(\omega_{1k}-\omega_{10k})}\right]$$

$$A_{2k} = \left[\frac{t_d^2\left(a_k\omega_k^7 + b_k\omega_k^5 + c_k\omega_k^4 + d_k\omega_k^3 + e_k\omega_k^2 + f\omega_k\right) + a_k^*\omega_k^9 + b_k^*\omega_k^7 + c_k^*\omega_k^6 + d_k^*\omega_k^5 + e_k^*\omega_k^4 + f_k^*\omega_k^3 + g_k^*\omega_k^2 + h_k^*\omega_k}{(\omega_{2k}-\omega_{1k})(\omega_{2k}-\omega_{3k})(\omega_{2k}-\omega_{4k})(\omega_{2k}-\omega_{5k})(\omega_{2k}-\omega_{6k})(\omega_{2k}-\omega_{7k})(\omega_{2k}-\omega_{8k})(\omega_{2k}-\omega_{9k})(\omega_{2k}-\omega_{10k})}\right]$$

$$A_{3k} = \left[\frac{t_d^2\left(a_k\omega_k^7 + b_k\omega_k^5 + c_k\omega_k^4 + d_k\omega_k^3 + e_k\omega_k^2 + f\omega_k\right) + a_k^*\omega_k^9 + b_k^*\omega_k^7 + c_k^*\omega_k^6 + d_k^*\omega_k^5 + e_k^*\omega_k^4 + f_k^*\omega_k^3 + g_k^*\omega_k^2 + h_k^*\omega_k}{(\omega_{3k}-\omega_{1k})(\omega_{3k}-\omega_{2k})(\omega_{3k}-\omega_{4k})(\omega_{3k}-\omega_{5k})(\omega_{3k}-\omega_{6k})(\omega_{3k}-\omega_{7k})(\omega_{3k}-\omega_{8k})(\omega_{3k}-\omega_{9k})(\omega_{3k}-\omega_{10k})}\right]$$

$$A_{4k} = \left[\frac{t_d^2\left(a_k\omega_k^7 + b_k\omega_k^5 + c_k\omega_k^4 + d_k\omega_k^3 + e_k\omega_k^2 + f\omega_k\right) + a_k^*\omega_k^9 + b_k^*\omega_k^7 + c_k^*\omega_k^6 + d_k^*\omega_k^5 + e_k^*\omega_k^4 + f_k^*\omega_k^3 + g_k^*\omega_k^2 + h_k^*\omega_k}{(\omega_{4k}-\omega_{1k})(\omega_{4k}-\omega_{2k})(\omega_{4k}-\omega_{3k})(\omega_{4k}-\omega_{5k})(\omega_{4k}-\omega_{6k})(\omega_{4k}-\omega_{7k})(\omega_{4k}-\omega_{8k})(\omega_{4k}-\omega_{9k})(\omega_{4k}-\omega_{10k})}\right]$$

$$A_{5k} = \left[\frac{t_d^2\left(a_k\omega_k^7 + b_k\omega_k^5 + c_k\omega_k^4 + d_k\omega_k^3 + e_k\omega_k^2 + f\omega_k\right) + a_k^*\omega_k^9 + b_k^*\omega_k^7 + c_k^*\omega_k^6 + d_k^*\omega_k^5 + e_k^*\omega_k^4 + f_k^*\omega_k^3 + g_k^*\omega_k^2 + h_k^*\omega_k}{(\omega_{5k}-\omega_{1k})(\omega_{5k}-\omega_{2k})(\omega_{5k}-\omega_{3k})(\omega_{5k}-\omega_{4k})(\omega_{5k}-\omega_{6k})(\omega_{5k}-\omega_{7k})(\omega_{5k}-\omega_{8k})(\omega_{5k}-\omega_{9k})(\omega_{5k}-\omega_{10k})}\right]$$

$$A_{6k} = \left[\frac{t_d^2\left(a_k\omega_k^7 + b_k\omega_k^5 + c_k\omega_k^4 + d_k\omega_k^3 + e_k\omega_k^2 + f\omega_k\right) + a_k^*\omega_k^9 + b_k^*\omega_k^7 + c_k^*\omega_k^6 + d_k^*\omega_k^5 + e_k^*\omega_k^4 + f_k^*\omega_k^3 + g_k^*\omega_k^2 + h_k^*\omega_k}{(\omega_{6k}-\omega_{1k})(\omega_{6k}-\omega_{2k})(\omega_{6k}-\omega_{3k})(\omega_{6k}-\omega_{4k})(\omega_{6k}-\omega_{5k})(\omega_{6k}-\omega_{7k})(\omega_{6k}-\omega_{8k})(\omega_{6k}-\omega_{9k})(\omega_{6k}-\omega_{10k})}\right]$$

Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

$$A_{7k}=\left[\frac{t_d^2\left(a_k\omega_k^7+b_k\omega_k^5+c_k\omega_k^4+d_k\omega_k^3+e_k\omega_k^2+f\omega_k\right)+a_k^*\omega_k^9+b_k^*\omega_k^7+c_k^*\omega_k^6+d_k^*\omega_k^5+e_k^*\omega_k^4+f_k^*\omega_k^3+g_k^*\omega_k^2+h_k^*\omega_k}{(\omega_{7k}-\omega_{1k})(\omega_{7k}-\omega_{2k})(\omega_{7k}-\omega_{3k})(\omega_{7k}-\omega_{4k})(\omega_{7k}-\omega_{5k})(\omega_{7k}-\omega_{6k})(\omega_{7k}-\omega_{8k})(\omega_{7k}-\omega_{9k})(\omega_{7k}-\omega_{10k})}\right]$$

$$A_{8k}=\left[\frac{t_d^2\left(a_k\omega_k^7+b_k\omega_k^5+c_k\omega_k^4+d_k\omega_k^3+e_k\omega_k^2+f\omega_k\right)+a_k^*\omega_k^9+b_k^*\omega_k^7+c_k^*\omega_k^6+d_k^*\omega_k^5+e_k^*\omega_k^4+f_k^*\omega_k^3+g_k^*\omega_k^2+h_k^*\omega_k}{(\omega_{8k}-\omega_{1k})(\omega_{8k}-\omega_{2k})(\omega_{8k}-\omega_{3k})(\omega_{8k}-\omega_{4k})(\omega_{8k}-\omega_{5k})(\omega_{8k}-\omega_{6k})(\omega_{8k}-\omega_{7k})(\omega_{8k}-\omega_{9k})(\omega_{8k}-\omega_{10k})}\right]$$

$$A_{9k}=\left[\frac{t_d^2\left(a_k\omega_k^7+b_k\omega_k^5+c_k\omega_k^4+d_k\omega_k^3+e_k\omega_k^2+f\omega_k\right)+a_k^*\omega_k^9+b_k^*\omega_k^7+c_k^*\omega_k^6+d_k^*\omega_k^5+e_k^*\omega_k^4+f_k^*\omega_k^3+g_k^*\omega_k^2+h_k^*\omega_k}{(\omega_{9k}-\omega_{1k})(\omega_{9k}-\omega_{2k})(\omega_{9k}-\omega_{3k})(\omega_{9k}-\omega_{4k})(\omega_{9k}-\omega_{5k})(\omega_{9k}-\omega_{6k})(\omega_{9k}-\omega_{7k})(\omega_{9k}-\omega_{8k})(\omega_{9k}-\omega_{10k})}\right]$$

$$A_{10}=\left[\frac{t_d^2\left(a_k\omega_k^7+b_k\omega_k^5+c_k\omega_k^4+d_k\omega_k^3+e_k\omega_k^2+f\omega_k\right)+a_k^*\omega_k^9+b_k^*\omega_k^7+c_k^*\omega_k^6+d_k^*\omega_k^5+e_k^*\omega_k^4+f_k^*\omega_k^3+g_k^*\omega_k^2+h_k^*\omega_k}{(\omega_{10k}-\omega_{1k})(\omega_{10k}-\omega_{2k})(\omega_{10k}-\omega_{3k})(\omega_{10k}-\omega_{4k})(\omega_{10k}-\omega_{5k})(\omega_{10k}-\omega_{6k})(\omega_{10k}-\omega_{7k})(\omega_{10k}-\omega_{8k})(\omega_{10k}-\omega_{9k})}\right]$$

The superconductor and dot states are identical. The simple algebra leads the above expression (15) in the following from;

$$\left\langle\left\langle c_{1-k\downarrow}^{+};c_{1k\uparrow}^{+}\right\rangle\right\rangle=\frac{1}{2\pi}\sum_{i=1}^{10}\frac{A_{ik}}{(\omega-\omega_{ik})} \quad \text{.......... (16)}$$

Where, ω_i' are the poles (i=1 to 10) of Green's functions and represent the quasi-particle energy branches of electronic structure in S-DQD-S junction. The Green's function is related to the correlation function and imaginary part of Green's function provide spectral density of Cooper pair as above equation (16) represents correlation corresponding to bound Cooper pairs in S-DQD-S junction. The spectral density of Cooper pairs can be calculated from the above Green's function $\left\langle\left\langle c_{1-k\downarrow}^{+};c_{1k\uparrow}^{+}\right\rangle\right\rangle$ representing the probability amplitude of bound paired state of electron with momentum k, (spin up↑) and –k with (spin down↓) by using the relationship:

$$A_k(\omega)=-\frac{1}{\pi}\operatorname{Im}\left\langle\left\langle c_{1-k\downarrow}^{+};c_{1k\uparrow}^{+}\right\rangle\right\rangle \quad \text{.......... (17)}$$

Where, (I_m) stands for imaginary part of Green's function given by equation (16) and involves δ-functions. In order to solve these (delta) functions, we have considered lorentzian type of broadening of the Cooper pair spectral density peaks by using the relationship between δ-function and lorentzian function as follows:

$$\delta(\omega-\omega_i)=\frac{1}{\pi}\lim_{\Gamma\to 0}\frac{\Gamma}{\Gamma^2+(\omega-\omega_i)^2} \quad \text{.......... (18)}$$

The broadening factor Γ is taken to be independent of ω and k to avoid complicity at this stage. There for, finally, we get expression of spectral density of Cooper pair for S-DQD-S Josephson junction as follow;

$$A_{k(\omega)} = \frac{\Gamma}{(\pi)^2} \sum_{i=1,}^{10} \frac{A_{ik}}{\left((\Gamma)^2 + (\omega - \omega_i)^2\right)} \quad \text{.......... (19)}$$

We have analyzed above electronic spectral density of Cooper pairs as a function of various parameters of model Hamiltonian (equation1) performing numerical computation and employing MATLAB software for this purpose. The spectral density of Cooper pairs in (S-DQD-S) junction predict the nature of Josephson transport in such tunnel junction and clues for maximum Josephson supercurrent. In the preceding section we have discussed our numerical results.

As explained in the first section, the green's function is related to correlation function. Therefore, the superconducting order parameter can be obtained from the corresponding Green's function with the help of standard procedure [5-6, 26]. Finally, we obtain the expression for superconducting order parameter for (S-QD-S) junction having a single level correlated (QD) as;

$$\Delta^{+} = -\frac{\Delta^{+}}{N} \sum_{k} \left[\begin{array}{l} \frac{A_{1k}}{e^{\beta' \omega_{1k}}+1} + \frac{A_{2k}}{e^{-\beta' \omega_{2k}}+1} + \frac{A_{3k}}{e^{\beta' \omega_{3k}}+1} + \frac{A_{4k}}{e^{-\beta' \omega_{4k}}+1} + \frac{A_{5k}}{e^{\beta' \omega_{5k}}+1} + \\ \frac{A_{6k}}{e^{-\beta' \omega_{6k}}+1} + \frac{A_{7k}}{e^{\beta' \omega_{7k}}+1} + \frac{A_{8k}}{e^{-\beta' \omega_{8k}}+1} + \frac{A_{9k}}{e^{\beta' \omega_{9k}}+1} + \frac{A_{10k}}{e^{-\beta' \omega_{10k}}+1} \end{array} \right] \quad \text{.......... (20)}$$

Where, $\beta' = \frac{1}{k_B T}$

Using above Eq.(20), one can estimate superconducting order parameters numerically in a self-consistent way by replacing summation over k value by an integral with cut-off energy $\pm\ \omega_c$ and a constant density of states around the Fermi level [7, 27].

Here, we are interested in the Josephson super current across the (S-QD-S) junction. Superconducting order parameter depends on temperature and various parameters of model Hamiltonian. Analyze the temperature dependence of Josephson super current for superconducting QD junction. We use the Ambegaokar-Baratoff formalism [22, 23] which connects the superconducting order parameter with Josephson super current applicable for Josephson tunnel junction as;

$$I_c R_n = \frac{\pi \Delta(T)}{2e} \tanh\left\{\frac{\Delta(T)}{2K_B T}\right\} \quad \text{.......... (21)}$$

Where, I_c is Josephson super current and R_n is junction resistance in normal state, At T=0, so the above equation reduces into the following form;

$$I_{co} R_n = \frac{\pi \Delta(O)}{2e} \quad \text{.......... (22)}$$

Where, $\Delta(0)$ and $\Delta(T)$ are superconducting order parameter at T=0 and finite temperature T< (T_c) can obtain $\Delta(0)$ from Eq. (17) at T=0K, Using Eqs. (18) and (19) the renormalized Josephson super current can be expressed as;

$$\frac{I_c}{I_{c0}} = \frac{\Delta(T)}{\Delta(0)} \tanh\left\{\frac{\Delta(T)}{2K_B T}\right\} \quad \text{.......... (23)}$$

One can analyze the renormalized Josephson supercurrent as a function of temperature and various parameters of the model Hamiltonian by numerical computation of $\Delta(0)$ and $\Delta(T)$ for the given set of physical different parameter all graph (2-6).

Now performed numerical computation using Eqs. (17) and (20) and as a first step the variation of $\Delta(T)/\Delta(0)$ vs (T/T_c) is very sharp curve different values of the QD energy level (ε) and keeping V=0.01eV and taking cut-off energy ω_c =0.023eV one can analyzed the role of quantum dot energy level on the superconducting order parameter and its temperature dependence. When ε increases so the superconducting order parameter also increases but for ε =0.002eV.

There is a cross over in the variation of superconducting order parameter vs. T / T_c now further increasing the energy of the dot state (ε =0.002eV) so the superconducting order parameter increases.

We have study of the dots energy level (ε) on the renormalized Josephson supercurrent $(I / I_c) vs (T / T_c)$ keeping V=0.01eV and ω_c =0.023eV fixed. So the energy level of the dots state. The renormalized Josephson super current increase in accordance with the theoretical analysis of the Josephson current across the junction (19).

Finally, it can be concluded that Josephson super current in S-QD-S tunnel junction interface with s-wave superconductors depend on the dot level energy, and spectral density double coupled quantum dots. The coupling parameter and Josephson Cooper pair tunnelling in an essential way. It will be interesting to extend this analysis by including on dots coulomb interaction so the Josephson resonance tunnelling and spin flip parameter into the model Hamiltonian.

In conclusion, Double level quantum dot weakly coupled to two superconducting lead left and right. We propose a new mechanism for a Josephson coupling between the leads that is qualitatively different from earlier proposals based on higher-order tunnelling processes via virtual states. Our proposal relies on generating a finite non-equilibrium pair amplitude on the dot by applying a bias voltage between normal and superconducting leads.

ACKNOWLEDGMENTS

One of the authors, Dr. Rajendra Kumar is Working as the Associate Professor (Physics) SDPG, College, Muzaffarnagar (U.P), India.

REFERENCES

[1] M.A.Reed,Quantum Dots, Scientific American 268, 118,(1993).
[2] A.Dhyani, B.S.Tewari, Ajay, Physica E, 42 162, (2009) .
[3] H. I. Jørgensen, T. Novotny, K. Grove-Rasmussen, K. Flensberg, and P. E. Lindelof, Nano Lett. 7, 2441 (2009) .
[4] A van Dam, Yu. V. Nazarov, E. P. A. M. Bakkers, S. De Franceschi, and L. P.Kouwenhoven, Nature 442, 667 (2006) .
[5] Meng T, Simon P and Florens S . Phys. Rev. B 79, 224521 (2009) .
[6] D.N. Zubarev, Usp. Fiz.Nauk, 71 (1960) .
[7] P.Ahalawat,P.K.Pathak,Ajay,Physica C , 455 ,46 (2007) .
[8] Vecino E, Mart´ın-Rodero A and Levy Yeyati A . Phys. Rev. B 68, 035105 (2003) .
[9] Affleck,I., Luitz D and Assaad F (2010) Phys. Rev. B 81 024509 (2010).
[10] L. Sohn, L.P. Kouwenhoven, G. Schon (Eds.), Mesoscopic Electron Transport, Kluwer, Dordrecht, (1997).
[11] L.I. Glazman, K.A. Matveev, Pis'mZ. JETP Lett. 49 , 659 (1989) .
[12] B.I. Spivak, S.A. Kivelson, Phys. Rev. B 43 , 3740 (1991) .
[13] C.B. Whan, T.P. Orlando, Phys. Rev. B 54, 5255 (1996) .
[14] A. Levy Yeyati, J.C. Cuevas, A. Lopez-Davalos, A.Martin-Rodero, Phys. Rev. B 55 R6137 (1997) [13] D.C. Ralph, C.T. Black, M. Tinkham, Phys. Rev. Lett. 74, 3241 (1995) .
[15] K.A. Matveev, et al., Phys. Rev. Lett. 70 , 2940 (1993).
[16] R. Bauernschmitt, J. Siewert, A. Odintsov, Yu.V. Nazarov,Phys. Rev. B 49, 4076 (1994) .
[17] R. Fazio, F.W.J. Hekking, A.A. Odintsov, Phys. Rev. B 53 , 6653 (1996).
[18] J. Siewert, G. Schon, Phys. Rev. B 54, 7424 (1996).
[19] S. Ishizaka, J. Sone, T. Ando, Phys. Rev. B 52, 8358 (1995).
[20] A.V. Rozhkov, D.P. Arovas, Phys. Rev. Lett. 82, 2788 (1999).
[21] J.-S. Caux, and A. Zagoskin, Phys. Rev. B 62, 1433 (2000).
[22] Anderson, P.W, Phys. Rev. 124, 41 (1961).
[23] Andreev, A. Sov. Phys. JETP 19, 1228 (1964) .
[24] J. M. Golden and B. I. Halperin, Phys. Rev.B 53, 3893 (1996).
[25] K. Flensberg, Phys. Rev.B 48, 11156 (1993).
[26] K. A. Matveev, Phys. Rev.B 51, 1743 (1995) .
[27] V. Ambegaokar,A.Baratoff,Phys.Rev.Lett.10, 486 (1963).
[28] Ding Guo-hui Chin Phys. Lett. 24, 2926 (2007).
[29] Y. Nisikawa, A. Oguri, J. of Physics-Conference Series 150 , 022066 (2009).
[30] R. Taranko, T. Kwapinski, Physica E,48, 157 (2013).

Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

[31] A. Dhyani, B.S. Twari, Ajay Physica E 41, 1179 (2009) .
[32] C. Karresch, S. Andergassen Phys. Rev B 84, 134512 (2011).
[33] R. Zitko, J. Mravlje, K. Haul. Phys. Rev. Lett 108, 066602 (2012).
[34] K.J. Pototzky and E.K.U. Gross: Cond-mat,arXiv:1405.5984v1 (2014),
[35] Wei-Cheng Lee, W. Park, Hemood Z. Arham :Cond-mat.,arXiv:1405.6357 (2014),
[36] Wei-Cheng Lee, W. Park, Hemood Z. Arham :Cond-mat.,arXiv: (2019),

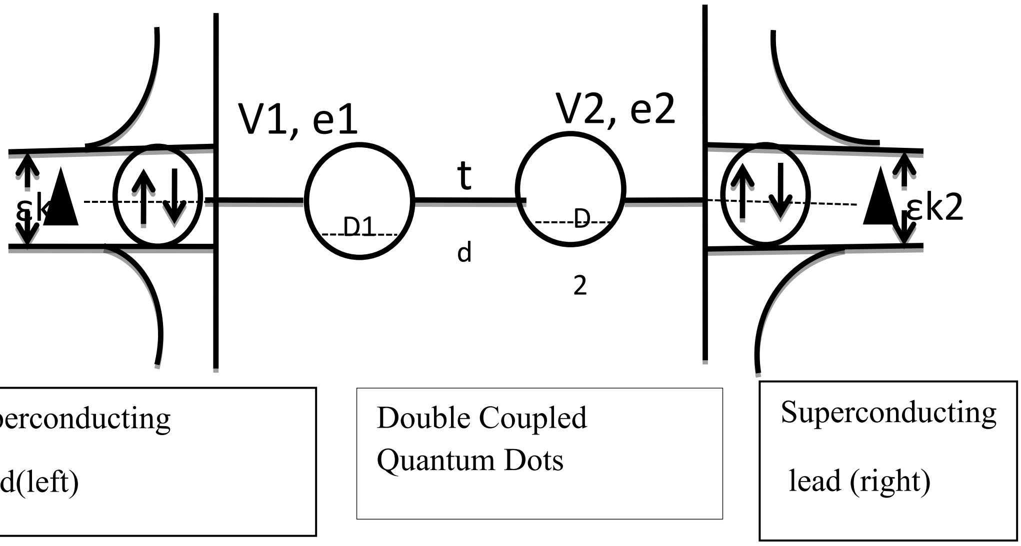

Fig 1 Schematic presentation of superconducting QD Josephson junction Electronic Spectral Density and Josephson current in a Double Coupled Quantum Dots Sandwich between Superconducting leads

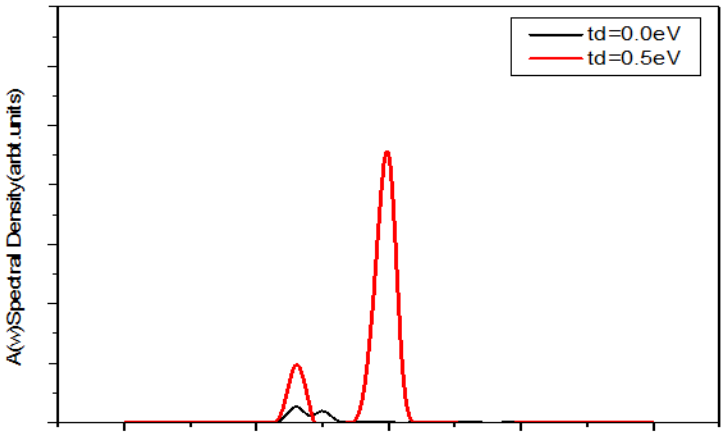

Figure 2

Stallion Journal for Multidisciplinary Associated Research Studies
ISSN (Online): 2583-3340
Volume-3 Issue-5 || October 2024 || PP. 244-254 https://doi.org/10.55544/sjmars.3.5.18

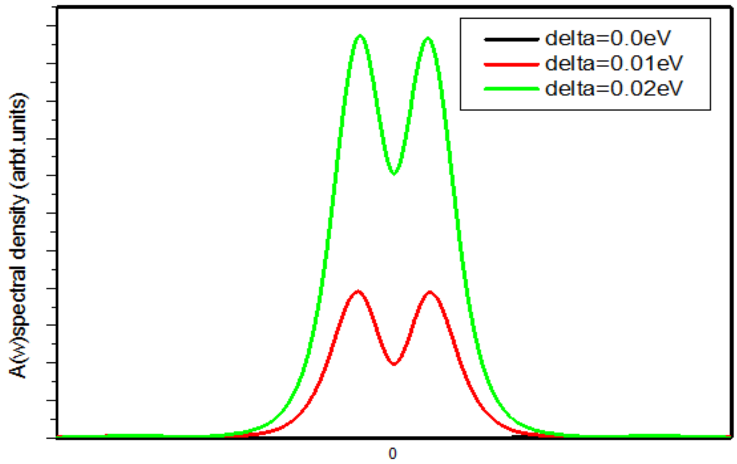

Figure 3

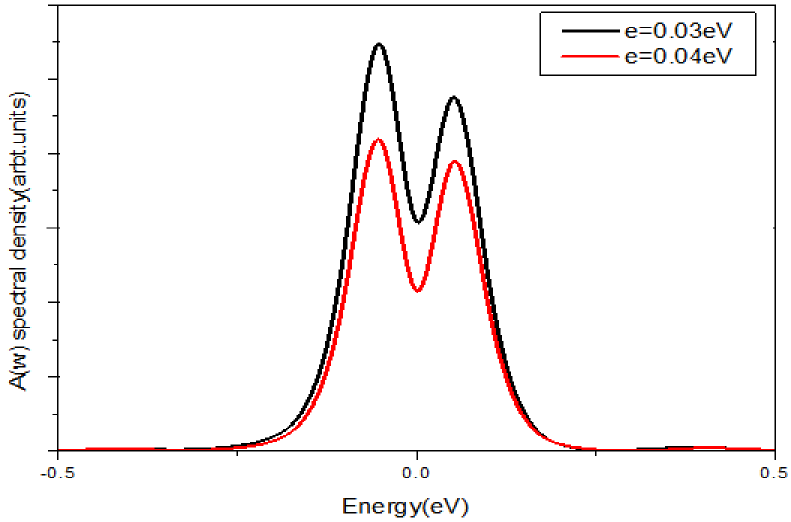

Figure 4

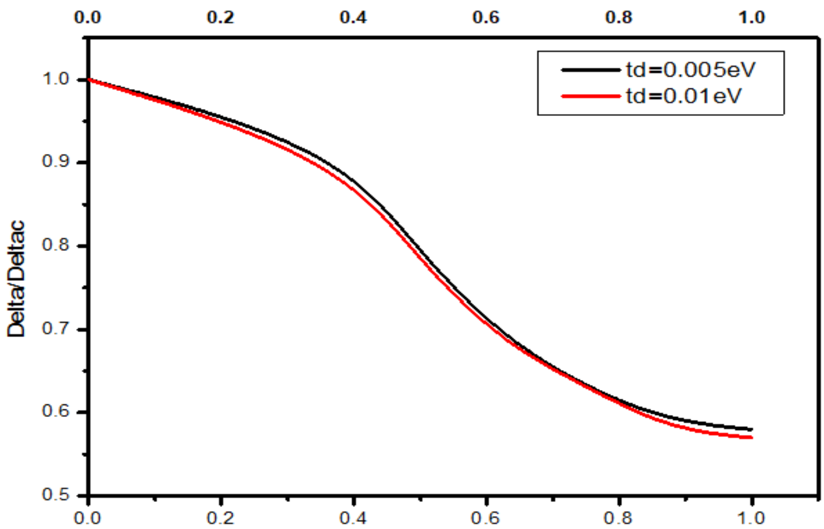

Figure 5

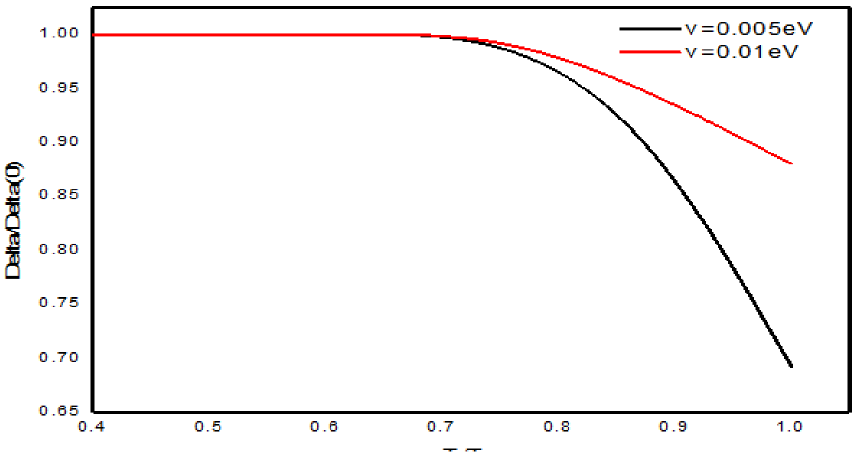
v=0.005eV
v=0.01eV
Delta/Delta(0)
1.00
0.95
0.90
0.85
0.80
0.75
0.70
0.65
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure 6