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A Generalized M/M/1 Queue with Mixture Service Times: Modeling and Analysis

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ABSTRACT

Queuing theory plays an important role in modeling congestion phenomena in service systems such as telecommunications, transportation, healthcare, and computer networks. Classical queuing models often assume exponential service time distributions; however, in many real-world situations service times may follow more complex distributions. In this study, a single-server queuing model with Poisson arrival process and a mixture service time distribution is considered. The proposed model, denoted as M/Mix/1, extends the classical M/M/1 framework by allowing the service time to follow a mixture distribution. The theoretical structure of the model is described, and key distributional properties such as mean, variance, moment generating function, cumulative distribution function, and Laplace transform are discussed. The results demonstrate that mixture distributions provide a flexible approach for modeling heterogeneous service mechanisms in queuing systems.

Keywords- M/M/1 Queue, Mixture Service Time Distribution, Queueing Theory, Stochastic Processes, Markov Chains.

I. INTRODUCTION

Queuing theory provides mathematical tools to analyze systems in which customers arrive to receive service from one or more servers. Such systems are commonly found in many real-world applications including banks, hospitals, manufacturing systems, computer networks, and telecommunication systems.

One of the most widely used queuing models is the **M/M/1 model**, where arrivals follow a Poisson process and service times follow an exponential distribution. Although this assumption simplifies analysis, it may not always represent real service processes accurately. In many practical situations, service time may arise from a **combination of different distributions**, reflecting heterogeneity in service mechanisms.

To address this limitation, researchers have proposed models in which the service time distribution is replaced by a **mixture distribution**. A mixture distribution allows the service process to be represented as a combination of multiple probability distributions, providing greater flexibility in modeling variability.

In this paper, we study a **single-server queuing system with Poisson arrivals and mixture service time distribution**, referred to as the **M/Mix/1 model**. The objective is to describe the structure of the model and examine important statistical properties of the service distribution that influence the performance of the queuing system.

II. DESCRIPTION OF THE QUEUING MODEL

2.1 System Characteristics

The queuing system considered in this study has the following characteristics:

- **Queue Type:** Single-server system

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- **Arrival Process:** Poisson process
- **Service Time Distribution:** Mixture distribution
- **Queue Discipline:** First-Come First-Served (FCFS)

This model can be represented using Kendall's notation as:

M/Mix/1

where:

- (M) indicates **Markovian (Poisson) arrivals**
- (Mix) indicates **mixture service time distribution**
- (1) indicates **a single server**

III. MATHEMATICAL EXPRESSIONS

a. Arrival Process (Poisson)

Definition: Customers arrive randomly over time such that the number of arrivals in a fixed interval follows a Poisson distribution.
Parameter: (λ) (average arrival rate, customers per unit time)

b. Service Time Distribution Mixture

Service: Mixture

SERVICE

$$h(t)=\alpha\left(\frac{l_n(\theta)}{\theta^t}\right)+(1-\alpha)\left(\mu e^{-\mu t}\right)\ 0<\alpha<1$$

$$h(s)=\alpha\frac{l_n(\theta)}{s+l_n(\theta)}+\frac{(1-\alpha)\mu}{s+\mu}$$

$$h\left(\lambda^*-\lambda^*z\right)=\frac{\alpha l_n(\theta)}{\lambda^*-\lambda^*z+l_n(\theta)}+\frac{(1-\alpha)\mu}{\lambda^*-\lambda^*z+\mu}$$

$$p_n=\alpha\frac{l_n(\theta)-\lambda^*}{l_n(\theta)}+(1-\alpha)(1-P)P^n\quad n\geq 1$$

Queuing System with Poisson and $\frac{l_n(\theta)}{\theta^t}$ series time distribution.

Model-1

Arrival: Poisson
Arrival rate=r^*
Service: Log

$$h(t)=\frac{l_n(\theta)}{\theta^t}:\theta>0,t>0$$

with Mean

$$E(t)=\int_0^\infty h(t)\,dt$$

$$=\int_0^\infty t\left[\frac{l_n(\theta)}{\theta^t}\right]dt$$

$$=l_n(\theta)\left[\frac{t(-\theta)^{-t}}{l_n(\theta)}\right]_0^\infty$$

$$E(t)=\left[\frac{1}{l_n(\theta)}\right]$$

Variance

$$V(t)=E\left(t^2\right)-\left[E(t)\right]^2$$

$$V(t)=\int_0^\infty t^2\left[\frac{l_n(\theta)}{\theta^t}\right]dt$$

$$=l_n(\theta)\left[\frac{t^2\left(-\theta^{-t}\right)}{l_n(\theta)}\right]_-^\infty+\left[\int_0^\infty\frac{t\left(-\theta^{-t}\right)}{l_n(\theta)}\right]^2 l_n(\theta)\,dt$$

$$E(t)^2=2\left(\frac{1}{l_n(\theta)}\right)^2$$

$$V(t)=2\left(\frac{1}{l_n(\theta)}\right)^2-\left(\frac{1}{l_n(\theta)}\right)^2 \qquad =\frac{1}{\left(l_n(\theta)\right)^2}$$

M.G.F

$$M_x(t)=E\left(e^{xt}\right)$$

$$=\int_0^\infty e^{tx}\left[\frac{l_n(\theta)}{`e^t}\right]dt$$

$$M.G.F.=l_n(\theta)\left[l_n\left(e^t\theta\right)\right]$$

$$C.F.=E\left[e^{itx}\right]$$

$$=\int_0^\infty e^{itx}\left[\frac{l_n(\theta)}{\theta}\right]dt \;=l_n(\theta)\left[l_n\left(e^{it}(\theta)\right)\right]$$

C.D.F.

$$F(t)=\int_0^t g(t)\,dt=\int_0^t\left[\frac{l_n(\theta)}{\theta^t}\right]dt=l_n(\theta)\left[\frac{\left(-\theta^{-t}\right)}{l_n(\theta)}\right]_0^t$$

$$F(t)=\left(1-\theta^{-t}\right)$$

$$R(t)=\int_t^\infty g(t)\,dt=1-\left(1-\theta^{-t}\right)$$

$$R(t)=\theta^{-t}$$

Conception Rate Function:

$$W(t)=\frac{g(t)}{S(t)}$$

$$=\frac{\frac{l_n\theta}{\theta^t}}{\frac{1}{\theta^t}}=l_n\theta$$

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Laplace transfer of $h(t)$

$$h(s)=L\left[h(t)\right]=\int_0^\infty e^{-st}h(t)\,dt$$

$$\therefore L\left(e^{at}\right)=\frac{1}{s-a}$$

$$\therefore \theta^{-t}=e^{l_n\left(\theta^{-t}\right)}=e^{-t.l_n(\theta)}=e^{-l_n(\theta)^t}$$

$$L\left(\theta^t\right)=L\left(e^{-l_n(\theta).t}\right)$$

$$=\frac{1}{S+l_n(\theta)}$$

$$=l_n(\theta)\int_0^\infty \frac{e^{-st}}{\theta^t}dt=l_n(\theta).\frac{1}{S+l_n(\theta)}=\frac{l_n(\theta)}{S+l_n(\theta)}$$

$$h(t)=\frac{l_n(\theta)}{s+l_n(\theta)}$$

$$h(s)=\frac{l_n(\theta)}{s+l_n(\theta)}$$

Mean Series

$$=\frac{1}{l_n(\theta)}$$

$$\rho=\lambda^*.\frac{1}{l_n(\theta)}$$

$$h\left(\lambda^*-\lambda^*\right)=\frac{l_n(\theta)}{\lambda^*-\lambda^*z+l_n(\theta)}$$

$$P(z)=\frac{(1-\rho)(z-1)h\left(\lambda^*-\lambda^*z\right)}{z-h\left(\lambda^*-\lambda^*z\right)}$$

$$=\frac{\left(1-\dfrac{\lambda^*}{l_n(\theta)}\right)(z-1).\dfrac{l_n(\theta)}{\lambda^*-\lambda^*z+l_n(\theta)}}{z-\dfrac{l_n(\theta)}{\lambda^*-\lambda^*z+l_n(\theta)}}=\left[\frac{\left(\alpha-\lambda^*\right)(z-1)}{z\left(\lambda^*-\lambda^*z+\alpha\right)-\alpha}\right]\qquad \therefore \alpha=l_n(\theta)$$

$$=\left(\alpha-\lambda^*\right)(z-1)\left[\left(\lambda^*-\lambda^*z+a\right)(z-\alpha)\right]^{-1}$$

$$=\left[\lambda^*(1-z)z+\alpha(z-1)\right]^{-1}=(1-z)^{-1}\left(\lambda^*z-\alpha\right)^{-1}$$

$$=\left(\alpha-\lambda^*\right)(z-1)(1-z)^{-1}\left(\lambda^*z-\alpha\right)^{-1}$$

$$=\left(\alpha-\lambda^*z\right)\left[\alpha\left(1-\frac{\lambda^*}{\alpha}z\right)\right]^{-1}$$

$$=\left(\frac{\alpha-\lambda^*}{\alpha}\right)+\lambda^*\frac{\left(\alpha-\lambda^*\right)\lambda^{*2}}{\alpha^2}z^2+\ldots$$

$$= p + p.z + p_2 z^2 + \ldots$$

$$p_n = \frac{\left(a-\lambda^*\right)}{a^{n+1}} = \frac{\left(l_n(\theta)-\lambda^*\right)\lambda^{*n}}{\left(l_n(\theta)\right)^{n+1}}$$

$$p_{n+1} = \frac{\left(l_n(\theta)-\lambda^*\right)\lambda^{*n+1}}{\left(l_n(\theta)\right)^{n+2}}$$

$$\frac{p_{n+1}}{p_n} = \frac{\left(l_n(\theta)-\lambda^*\right)\lambda^{*n+1}}{\left(l_n(\theta)\right)^{n+2}} * \frac{l_n(\theta)^{n+1}}{\left(l_n(\theta)-\lambda^*\right)\lambda^{*n}}$$

$$\frac{\lambda^*}{l_n(\theta)}$$

$$p_{n+1} = \frac{\lambda^*}{l_n(\theta)} p_n \quad n \geq 1$$

$$p_0 = \frac{l_n(\theta)-\lambda^*}{l_n(\theta)}$$

$$p_n = \frac{\left(l_n(\theta)-\lambda^*\right)\lambda^{*n}}{l_n(\theta)^{n+1}} \quad n \geq 0$$

$$= \frac{l_n(\theta)-\lambda^*}{l_n(\theta)}\left(\frac{\lambda}{l_n(\theta)}\right)^n$$

$$L = \sum_{n=0}^{\infty} n \frac{\left(l_n(\theta)-\lambda^*\right)\lambda^{*n}}{l_n(\theta)^{n+1}}$$

$$= \left(l_n(\theta)-\lambda^*\right)\sum_{n=0}^{\infty} n \frac{\lambda^{*n}}{l_n(\theta)^{n+1}} \qquad = \frac{l_n(\theta)-\lambda^*}{l_n(\theta)}\sum_{n=0}^{\infty} n \frac{n\lambda^{*n}}{l_n(\theta)^{n+1}}$$

$$= \frac{l_n(\theta)-\lambda^*}{l_n(\theta)}\sum_{n=0}^{\infty} n\left(\frac{\lambda}{l_n(\theta)}\right)^n$$

$$= (1-\rho)\sum_{n=0}^{\infty} n\rho^n \qquad = (1-\rho)\left[\rho + 2\rho^2 + 3\rho^3 + 4\rho^4 + \ldots\right]$$

$$= (1-\rho).\frac{\rho}{(\rho-1)^2} \qquad = \frac{\rho}{1-\rho}$$

$$W = \frac{L}{\lambda^*}$$

$$L_1 = \sum_{n=1}^{\infty} (n-1)p_n = \sum_{n=1}^{\infty} (n-1)(1-\rho).\rho^n$$

$$= (1-\rho)\sum_{n=2}^{\infty} (n-1).\rho^n$$

$$(1-\rho).\frac{\rho^2}{(\rho-1)^2} = \frac{\rho^2}{1-\rho}$$

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$$W_q = \frac{l_r}{\lambda^*}$$

IV. APPLICATIONS OF THE MODEL

The M/Mix/1 queuing model can be applied in several practical scenarios including:

- Computer networks where different packets require different service times
- Hospitals where patients require varying levels of treatment
- Manufacturing systems with heterogeneous processing tasks
- Telecommunication systems handling different types of data traffic

The mixture service distribution provides flexibility in representing real-world service variability.

V. CONCLUSION

In this paper, a single-server queuing system with Poisson arrivals and mixture service time distribution was considered. The proposed M/Mix/1 model extends the traditional M/M/1 framework by incorporating heterogeneous service processes through mixture distributions. Key statistical properties such as mean, variance, moment generating function, cumulative distribution function, and Laplace transform of the service distribution were discussed. The mixture distribution provides a flexible and realistic representation of service mechanisms in many practical systems. The results demonstrate that such models can effectively be used in queuing systems where service times cannot be adequately described by a single probability distribution.

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