

Queuing System with Poisson Arrival Distribution and Mixture Service Time Distribution: An M/Mix/1 Model

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ABSTRACT

Queueing models play an important role in analyzing congestion phenomena in service systems such as telecommunication networks, computer systems, banks, hospitals, and transportation systems. Traditional queueing models often assume exponential service time distributions; however, real-life service processes frequently exhibit heterogeneous behavior that cannot be adequately captured by a single distribution. In this paper, we propose a single-server queueing model with a Poisson arrival process and a mixture service time distribution, referred to as the M/Mix/1 model. The mixture distribution allows the service mechanism to represent multiple service phases or heterogeneous service requirements. Key characteristics of the mixture distribution including its probability density function, cumulative distribution function, moment generating function, and variance are discussed. The proposed framework demonstrates how mixture distributions can enhance the flexibility of queueing models for realistic service systems.

Keywords- Queueing Theory, Markovian Queues, Random Memory Queues, Stochastic Processes, Transition Probabilities, Queue Characteristics.

I. INTRODUCTION

Queueing theory is a fundamental area of applied probability that studies the behavior of waiting lines formed by customers or jobs requiring service. These models help analyze system performance measures such as waiting time, queue length, and system utilization. Classical queueing systems such as M/M/1, M/G/1, and G/G/1 assume specific distributions for arrivals and service times.

In many practical situations, the assumption of a single service time distribution may not adequately represent real systems. For example, service operations in hospitals, banking systems, computer networks, and call centres often involve multiple service stages or heterogeneous customer types, leading to variability that cannot be captured by a single exponential or deterministic distribution.

To address this limitation, mixture distributions can be used to model service times. A mixture distribution combines two or more component distributions with certain probabilities, allowing the service mechanism to represent different service modes within a single framework.

In this paper, we study a single-server queueing system with Poisson arrivals and mixture service time distribution, referred to as the M/Mix/1 queueing model. The purpose of this study is to explore the applicability of mixture distributions in queueing systems and discuss the mathematical characteristics of the proposed model.

II. MATHEMATICAL EXPRESSIONS

a. Arrival Process (Poisson)

Definition: Customers arrive randomly over time such that the number of arrivals in a fixed interval follows a Poisson distribution.

Parameter: (λ) (average arrival rate, customers per unit time)

b. Service Time Distribution Mixture

Service: Mixture

$$h(t) = \alpha \left(\frac{l_n(\theta)}{\theta^t} \right) + (1-\alpha) (\mu e^{-\mu t}) \quad 0 < \alpha < 1$$

$$\begin{aligned} E(t) &= \int_0^\infty t h(t) dt \\ &= \int_0^\infty t \left(\alpha \frac{l_n(\theta)}{\theta^t} + (1-\alpha) \mu e^{-\mu t} \right) dt \\ &= \alpha l(\theta) \left[\frac{t(-\theta)^t}{l_n(\theta)} \right]_0^\infty + \left[\int_0^\infty \frac{1 \cdot (\theta^{-t})}{l_n(\theta)} \alpha l_n(\theta) dt \right] + (1-\alpha) \mu \left[\frac{t(e)^{-\mu t}}{-\mu} \right]_0^\infty \\ &\quad + \left[\int_0^\infty \frac{e^{-\mu t}}{\mu} (1-\alpha) \mu dt \right] \end{aligned}$$

$$E(t) = \alpha \left[\frac{1}{l_n(\theta)} \right] + (1-\alpha) \left(\frac{1}{\alpha} \right)$$

Variance:

$$\begin{aligned} V(t) &= E(t^2) - (E(t))^2 \\ E(t^2) &= \int_0^\infty t^2 \left(\frac{\alpha l_n(\theta)}{\theta^t} + (1-\alpha) \mu e^{-\mu t} \right) dt \\ &= \alpha l_n(\theta) \left[\frac{t^2(-\theta^{-t})}{l_n(\theta)} \right]_0^\infty + \left[\int_0^\infty \frac{t(\theta^{-t})}{l_n(\theta)} 2 < l_n(\theta) \right] + (1-\alpha) \mu \left[\frac{t^2(\theta)^{-\mu t}}{-\mu} \right]_0^\infty \end{aligned}$$

$$E(t^2) = 2\alpha \left[\frac{1}{l_n(\theta)} \right] + 2(1-\alpha) \left(\frac{1}{\alpha} \right)^2$$

$$\begin{aligned} V(t) &= 2\alpha \left[\frac{1}{l_n(\theta)} \right] + 2(1-\alpha) \left(\frac{1}{\alpha} \right)^2 - \left[\alpha \frac{1}{l_n(\theta)} + (1-\alpha) \left(\frac{1}{\alpha} \right) \right]^2 \\ &= \frac{(1-\alpha)^2}{\mu^2} + \frac{\alpha(2-\alpha)}{l_n(\theta)^2} - \frac{2\alpha(1-\alpha)}{\mu l_n(\theta)} \end{aligned}$$

M.G.F.

$$\begin{aligned} M_\alpha(t) &= E(e^{\alpha t}) \\ &= \int_0^\infty e^{t x} \left[\alpha \frac{l_n(\theta)}{\theta^t} + (1-\alpha) \mu e^{-\mu t} \right] \\ &\quad \alpha l_n(\theta) \left[l_n(e^t(\theta)) + (1-\alpha) \frac{dt}{t-\mu} \right] \end{aligned}$$

$$\begin{aligned}
 C.F &= E(e^{ix}) \\
 &= \int_0^\infty e^{itx} \left[\frac{\alpha l^n(\theta)}{\theta^t} + (1-\alpha)\mu e^{-\mu t} \right] dt \\
 &= \alpha l_n(\theta) [l_n | e^{it}] + (1-\alpha) \left(\frac{dt}{it - \mu} \right)
 \end{aligned}$$

C.D.F.

$$\begin{aligned}
 F(t) &= \int_0^t g(t) dt \\
 &= \int_0^t \left[\alpha \frac{l_n(\theta)}{\theta^t} + (1-\alpha)\mu e^{-\mu t} \right] dt \\
 &= \alpha l_n(\theta) \left[\frac{-\theta^t}{l_n(\theta)} \right]_0^t + (1-\alpha) \left[\frac{e^{-\mu t}}{-\mu} \right]_0^t \\
 F(t) &= \alpha(1-\theta^t) + (1-\alpha)[1 - e^{-\mu t}]
 \end{aligned}$$

Atrilrt function.

$$\begin{aligned}
 R(t) &= \int_t^\infty h(t) dt \\
 &= 1 - \alpha(1-\theta^{-t}) - (1-\alpha)(1 - e^{-\mu t}) \\
 R(t) &= \alpha\theta^{-t} + (1-\theta)e^{-\mu t}
 \end{aligned}$$

Conception Rate Function

$$\begin{aligned}
 \dots &= \frac{g(t)}{S(t)} \\
 &= \left[\frac{\alpha \frac{l_n(\theta)}{\theta^t} + (1-\alpha)\mu e^{-\mu t}}{\alpha\theta^{-t} + (1-\alpha)\mu e^{-\mu t}} \right] \\
 h(t) &= \alpha \left[\frac{l_n(\theta)}{\theta^t} \right] + (1-\alpha)\mu e^{-\mu t} \quad 0 < \alpha < 1 \\
 h(s) &= \alpha \frac{l_n(\theta)}{s + l_n(\theta)} + \frac{(1-\alpha)\mu}{s + \mu} \\
 h(\lambda^\alpha - \lambda^\alpha) &= \alpha \frac{l_n(\theta)}{\lambda^\alpha - \lambda^\alpha z + l_n(\theta)} + \frac{(1-\alpha)\mu}{\lambda^\alpha - \lambda^\alpha z + \mu} \\
 P_n &= \alpha \frac{l_n(\theta) - \lambda^\alpha}{l_n(\theta)} + (1-\alpha)(1 - P_2)P_2^n \quad n \geq 1 \\
 \lambda^* . E(t) &= \frac{\lambda^* \alpha}{l_n(\theta)} + \frac{\lambda^* (1-\alpha)}{\mu} = \alpha P_1 + (1-\alpha)P_2 \\
 &= \frac{(1-P)(z-1)h(\lambda^* - \lambda^* z)}{z - h(\lambda^* - \lambda^* z)}
 \end{aligned}$$

$$\begin{aligned}
 &= \frac{(1-P)(z-1) \left(\frac{\alpha l_n(\theta)}{\lambda^* - \lambda^* z + l_n(\theta)} + \frac{(1-\alpha)\theta\mu}{\lambda^* - \lambda^* z + \mu} \right)}{z - \left(\frac{\alpha l_n(\theta)}{\lambda^* - \lambda^* z + l_n(\theta)} + \frac{(1-\alpha)\theta\mu}{\lambda^* - \lambda^* z + \mu} \right)} \\
 &= \frac{(1-P)(z-1)(\alpha(1-z)P_2 + 1) + (1-\alpha)(1-z)P_1 + 1)}{z((1-z)P_1 + 1)(1-z)P_2 + 1 - \alpha((1-z)P_2 + 1) - (1-\alpha)(1-z)P_1 + 1)} \\
 &= \frac{(\alpha P_1 + (1-\alpha)P_2)(z-1)(\alpha(1-z)P_2 + \alpha + (1-\alpha)(1-z)P_1 + (1-\alpha))}{z((1-z)^2 P_1 P_2 + (1-z)P_1 + (1-z)P_2 + 1) - (\alpha(1-z)P_2 + \alpha + (1-\alpha)(1-z)P_2 + (1-\alpha))} \\
 &= (\alpha P_1 + (1-\alpha)P_2)(z-1)(\alpha(1-z)P_2 + (1-\alpha)(1-z)P_1 + 1) \\
 &= (\alpha P_1 + (1-\alpha)P_2)(z-1)((1-z)(\alpha P_2 + (1-\alpha)P_1) + 1) \\
 &= (\alpha P_1 + (1-\alpha)P_2) \left[(z-1)^2 (-\{\alpha P_2 + (1-\alpha)P_1\}) + (z-1) \right] \\
 &= (z-1)^2 \left[\alpha^2 P_1 P_2 + \alpha(1-\alpha)P_1 + \alpha(1-\alpha)P_2 + (1-\alpha)^2 P_1 P_2 \right] + (z-1)(\alpha P_1 + (1-\alpha)P_2) \\
 &= (z-1)^2 \left[\alpha^2 + (1-\alpha)^2 P_1 P_2 + \alpha(1-\alpha)(P_1 + P_2) \right] + (z-1)(\alpha P_1 + (1-\alpha)P_2)
 \end{aligned}$$

MIX

$$\begin{aligned}
 L &= \frac{\alpha P_1}{1-P_1} + \frac{(1-\alpha)P_2}{1-P_2} \\
 &= \frac{\alpha P_1(1-P_2) + (1-\alpha)P_2(1-P_1)}{(1-P_1)(1-P_2)} \\
 &= \frac{\alpha(P_1 - P_2) + (1-P_1)P_2}{1 - (P_1 + P_2) + P_1 P_2}
 \end{aligned}$$

III. DISCUSSION

The use of mixture distributions provides a more flexible representation of service time behavior in real-life queueing systems. In practical service environments, customers often require different service durations depending on their service type, complexity of requests, or system conditions.

The mixture distribution allows the model to capture:

- heterogeneous customer service requirements
- multiple service phases
- varying service intensities

As a result, the proposed M/Mix/1 model can better approximate real-world systems compared to classical models that assume homogeneous service times.

IV. CONCLUSION

In this study, a queueing model with Poisson arrival distribution and mixture service time distribution has been proposed and discussed. The model represents a single-server system (M/Mix/1) in which service times follow a mixture distribution rather than a single distribution.

The mathematical properties of the mixture distribution, including mean, variance, moment generating function, and cumulative distribution function, were presented. The analysis shows that mixture distributions provide greater flexibility in modeling heterogeneous service mechanisms.

The proposed model demonstrates that mixture distributions can effectively be incorporated into queueing systems to capture real-world service variability. Future research may focus on deriving explicit performance measures, conducting simulation studies, and applying the model to practical systems such as communication networks, healthcare services, and banking operations.

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